Short-Range Nonlinear Feedback Strategies for Aircraft Pursuit-Evasion

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Prelinearizing transformations are combined with the linear quadratic pursuit-evasion game results to synthesize explicit nonlinear feedback strategies for aircraft pursuit-evasion in the endgame. Three-dimensional point-mass models for aircraft are employed in this work. Numerical results are given for two different engagement scenarios. Future applications include the derivation of collision avoidance strategies for aircraft, aircraft rendezvous, and guidance for coordinated maneuvering with other aircraft.

Introduction

AIRCRAFT pursuit-evasion problems have been of considerable interest in recent literature.[1–8] Although time for capture appears to be the accepted performance index in these problems, in the endgame it is reasonable to minimize the terminal miss in a given time. Previous research in this area employed forced singular perturbation theory[5–8] and linearized dynamics[9,10] for synthesizing implementable pursuit-evasion guidance laws. The present research is aimed at the solution to the endgame problem wherein the pursuer attempts to minimize the terminal miss, while the evader attempts to maximize it in a given time using nonlinear point-mass models for aircraft. The central ideas here are the use of linearizing transformations to cast the pursuit-evasion problem between two aircraft into a linear, time-invariant form, obtaining its solution using the available differential game results, and transforming the guidance scheme back to the original coordinates to obtain nonlinear feedback laws. It is important to note that the performance index considered in this framework can only contain the transformed states and controls. Further, the need for synthesizing a feedback law would require the performance index to be quadratic in transformed state and control variables. As will be clear in the ensuing, these factors do not unduly restrict the usefulness of the results.

The proposed approach involves the use of the well-known linear quadratic pursuit evasion game results[9,10] in conjunction with the recent theory of prelinearizing transformations.[11,12] The formulation uses the complete nonlinear aircraft point-mass model and assumes the availability of perfect information. The present approach can be termed as a nonlinear-quadratic (NQ) differential game. Since the guidance scheme employs a part of aircraft nonlinearities for feedback, it requires more information about the vehicle than previously reported schemes. However, it is to be emphasized that each participant in the game requires detailed information only about itself. Typically, the nature of thrust, drag, and lift are required.

Previous derivations of pursuit-evasion laws in the endgame[9,10,13] assume linear vehicle dynamics and as such, completely ignore the dynamic and kinematic nonlinearities inherent in realistic vehicle models. Note that one cannot employ conventional linearization via the Taylor-series expansion in a differential game setting because it is impossible to synthesize a nominal trajectory. The guidance laws emerging from the present analysis are identical to those from earlier approaches when the model is in prelinearized form. But, upon transforming back to the original coordinates, they become nonlinear, imbedding the vehicle nonlinearities in the feedback. As in Refs. 10 and 13, the present guidance law also requires an estimate of the time-to-go and an assessment of the participant's acceleration capabilities in an Earth-fixed frame.

Note that the guidance strategies presented here are useful only in short-range situations because they do not explicitly optimize the vehicle performance in terms of attempting to lead the adversary to the regions of superiority.

The analysis given in this paper can be extended for deriving collision avoidance laws between two cooperating aircraft, which should be useful in terminal area maneuvers for transport and civil aviation aircraft. This research is also useful for bank-to-turn missile guidance against maneuvering targets. Aerial rendezvous and coordinated maneuvering are two other problems that can be successfully addressed with the proposed approach. Following the methodology presented here, spacecraft guidance laws in the presence of arbitrary gravitational accelerations can also be derived.[14]

Aircraft Models and Prelinearization

The point-mass aircraft model for three-dimensional flight over a flat, nonrotating earth with the assumption of thrust along the path is given by

$$\dot{V} = \frac{T-D}{m} - g\sin\gamma \tag{1}$$

$$\dot{\gamma} = \frac{g}{V}(n\cos\phi - \cos\gamma) \tag{2}$$

$$\dot{\chi} = \frac{g}{V}\frac{n\sin\phi}{\cos\gamma} \tag{3}$$

$$\dot{x} = V\cos\gamma\cos\chi \tag{4}$$

$$\dot{y} = V\cos\gamma\sin\chi \tag{5}$$

$$\dot{h} = V\sin\gamma \tag{6}$$

$$T = T(h,M,\eta) \tag{7}$$

$$D = D(h,M,n) \tag{8}$$

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In these expressions, V is the airspeed, T the thrust, D the aerodynamic drag, m the mass, g acceleration due to gravity, γ the flight-path angle, χ the heading angle, x the downrange, y the crossrange, and h is the altitude. The load factor n, the bank angle ϕ, and the throttle η are the control variables in this model. It will be assumed here that if the required thrust were known, the throttle setting may be computed using a table look-up.

Note that the present formulation can handle thrust reversal as well as the effect of speed brakes, if these capabilities were available. The models for both the pursuer and the evader are in the same form, except for the differences in the functions T and D. Let the pursuer states be denoted by the subscript p and the evader states be denoted by e. The nonlinear aircraft models for the pursuer and evader may be transformed into the Brunovsky canonical form[11] by differentiating the expressions (4–6) with respect to time, and substituting for $\dot{V}$, $\dot{\gamma}$, and $\dot{\chi}$ from the expressions (1)–(3). Next, defining six pseudocontrol variables, the prelinearized aircraft models can be put in the form

$$\ddot{x}_p = U_1, \qquad \ddot{y}_p = U_2, \qquad \ddot{h}_p = U_3 \tag{9}$$

$$\ddot{x}_e = W_1, \qquad \ddot{y}_3 = W_2, \qquad \ddot{h}_e = W_3 \tag{10}$$

where U_1, U_2, U_3, W_1, W_2, and W_3 are the pseudocontrol variables given by

$$U_1 = \frac{(T_p - D_p)}{m_p} \cos\gamma_p \cos\chi_p - g n_p(\sin\gamma_p \cos\chi_p \cos\phi_p + \sin\chi_p \sin\phi_p) \tag{11}$$

$$U_2 = \frac{(T_p - D_p)}{m_p} \cos\gamma_p \sin\chi_p + g n_p(\cos\chi_p \sin\phi_p - \sin\gamma_p \sin\chi_p \cos\phi_p) \tag{12}$$

$$U_3 = \frac{(T_p - D_p)}{m_p} \sin\gamma_p + g n_p \cos\gamma_p \cos\phi_p - g \tag{13}$$

$$W_1 = \frac{(T_e - D_e)}{m_e} \cos\gamma_e \cos\chi_e - g n_e(\sin\gamma_e \cos\chi_e \cos\phi_e + \sin\chi_e \sin\phi_e) \tag{14}$$

$$W_2 = \frac{(T_e - D_e)}{m_e} \cos\gamma_e \sin\chi_e + g n_e(\cos\chi_e \sin\phi_e - \sin\gamma_e \sin\chi_e \cos\phi_e) \tag{15}$$

$$W_3 = \frac{(T_e - D_e)}{m_e} \sin\gamma_e + g n_e \cos\gamma_e \cos\phi_e - g \tag{16}$$

It may be noted that the vehicle models given by the expressions (9) and (10) are in a linear time invariant form. If the pseudocontrol variables U_1, U_2, U_3, and W_1, W_2, W_3 were known, the actual aircraft controls can be obtained using the following expressions:

$$\phi_p = \tan^{-1}\left[\frac{U_2 \cos\chi_p - U_1 \sin\chi_p}{\cos\gamma_p(U_3 + g) - \sin\gamma_p(U_1 \cos\chi_p + U_2 \sin\chi_p)}\right] \tag{17}$$

$$n_p = \frac{\cos\gamma_p(U_3 + g) - \sin\gamma_p(U_1 \cos\chi_p + U_2 \sin\chi_p)}{g \cos\phi_p} \tag{18}$$

$$T_p = [\sin\gamma_p(U_3 + g) + \cos\gamma_p(U_1 \cos\chi_p + U_2 \sin\chi_p)]m_p + D_p \tag{19}$$

$$\phi_e = \tan^{-1}\left[\frac{W_2 \cos\chi_e - W_1 \sin\chi_e}{\cos\gamma_e(W_3 + g) - \sin\gamma_e(W_1 \cos\chi_e + W_2 \sin\chi_{xe})}\right] \tag{20}$$

$$n_e = \frac{\cos\gamma_e(W_3 + g) - \sin\gamma_e(W_1 \cos\chi_e + W_2 \sin\chi_e)}{g \cos\phi_e} \tag{21}$$

$$T_e = [\sin\gamma_e(W_3 + g) + \cos\gamma_e(W_1 \cos\chi_e + W_2 \sin\chi_e)]m_e + D_e \tag{22}$$

All the terms on the right-hand side of the expressions (17)–(22) are state- and pseudocontrol-dependent. One may observe that prelinearization is accomplished here simply by writing the aircraft equations of motion in an Earth-fixed coordinate frame. Thus, physically, the pseudocontrol variables represent the acceleration components in the x, y, h coordinate system.

Pursuit-Evasion Games in Prelinearized Form

In order to motivate the development of the nonlinear feedback guidance scheme, this section will discuss a pursuit-evasion game in prelinearized coordinates. For notational convenience, let the prelinearized model for the pursuer and the evader be given by the following vector differential equations with V_p, $V_e \in \boldsymbol{R}^3$; X_p, $X_e \in \boldsymbol{R}^3$; U, $W \in \boldsymbol{R}^3$:

$$\dot{V}_p = U, \; V_p(t_0) \text{ given} \tag{23}$$

$$\dot{V}_e = W, \; V_e(t_0) \text{ given} \tag{24}$$

$$\dot{X}_p = V_p, \; X_p(t_0) \text{ given} \tag{25}$$

$$\dot{X}_e = V_e, \; X_e(t_0) \text{ given} \tag{26}$$

where the subscripts p and e stand for pursuer and evader, respectively. The control variables in the model are the acceleration components in the Earth-fixed coordinate system, the pseudocontrol variables in the transformed model. The pursuer uses the control U to capture the evader and the evader uses the control W to attempt to avoid capture.

In the endgame, the objective of the pursuer is to minimize the terminal miss, which the pursuer attempts to maximize. The terminal miss is defined as

$$\tfrac{1}{2} Z^T(t_f) S_f Z(t_f) \tag{27}$$

where

$$Z = X_p - X_e \tag{28}$$

The superscript T denotes the transpose of the vector, S_f is a positive semidefinite matrix, and the final time t_f is assumed to be known. The terminal miss may include the requirement that the pursuer and evader speeds be approximately equal at the termination time t_f. Alternately, one may require the pursuer to match the evader's speed throughout the game by including an integral quadratic term of the form

$$\int_{t_0}^{t_f} Y^T C Y \, dt \tag{29}$$

with

$$Y = V_p - V_e \tag{30}$$

in the performance index, C being a positive semidefinite matrix. This term, however, will not be included in the ensuing analysis. An integral quadratic control constraint is next imposed to make the game meaningful, and also for obtaining the pursuit-evasion guidance laws in feedback form. Thus

$$\int_{t_0}^{t_f} U^T A U \, dt \le a_p \tag{31}$$

$$\int_{t_0}^{t_f} W^T B W \, dt \le a_e \tag{32}$$

where A and B are positive definite matrices, and a_p, a_e are positive constants. Geometrically, the integrand of these constraints represents ellipsoids in the downrange, crossrange, altitude (x, y, h) frame. The matrices A and B can be time-varying and are chosen based on the maximum vehicle acceleration capability along the x, y, h directions and the remaining time. For example, if the pursuer had a maximum acceleration capability a_{max} along the x, y, h directions, the diagonal elements of the matrix A can be chosen as[10]

$$A(i,i) = 1/[a_{max}^2 (t_f - t_0)] \tag{33}$$

It is clear that both pursuer and evader will use all the control they can in the case of finite-minimax terminal miss, so that the control constraints will be equalities. Adjoining these constraints to the performance index, one has

$$J = \min_U \max_W \left[\frac{1}{2} Z^T(t_f) S_f Z(t_f) + \int_{t_0}^{t_f} [U^T A U - W^T B W] \, dt \right] \tag{34}$$

The saddle-point solution[10] to this problem is given by

$$U = -A^{-1} G^T S [ZY]^T \tag{35}$$

$$W = -B^{-1} G^T S [ZY]^T \tag{36}$$

where the gain matrix S is the solution of the matrix-Riccati equation

$$\dot{S} = -SF - F^T S + SG(A^{-1} - B^{-1}) G^T S \tag{37}$$

with the terminal boundary condition $S(t_f) = S_f$. The matrices F and G are given by

$$F = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \tag{38}$$

$$G = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{39}$$

Here 0 is a 3×3 zero matrix and I is a 3×3 identity matrix. This completes the solution of the pursuit-evasion game in prelinearized form. If A and B were constant and known, this problem may be solved in closed form.[15] Otherwise, the matrix-Riccati equation [Eq. (37)] will have to be integrated from the final to the initial time with the specified boundary conditions. In any case, this is a well-understood problem and several excellent papers are currently available. In the present research, however, the solution will be specialized to a particular case to gain further clarity in the nonlinear guidance law derivation. If S_f, A and B were constant diagonal matrices with $S_f(i,i) = s_i$, $A(i,i) = 1/a_i$, and $B(i,i) = 1/b_i$, with $i = 1, 2, 3$, the saddle-point solution[10] turns out to be

$$U = K_1(X_p - X_e) + K_2(V_p - V_e) \tag{40}$$

$$W = K_3 U \tag{41}$$

In the expressions (40) and (41), K_1, K_2, and K_3 are 3×3 diagonal matrices with

$$K_1(i,i) = \frac{-a_i t_{go}}{(1/s_i) + (a_i - b_i)[t_{go}^3/3]}, \quad i = 1, 2, 3 \tag{42}$$

$$K_2(i,i) = K_1(i,i) t_{go}, \quad i = 1, 2, 3 \tag{43}$$

$$K_3(i,i) = b_i / a_i, \quad i = 1, 2, 3 \tag{44}$$

and

$$t_{go} = t_f - t \tag{45}$$

This solution is in a feedback form and is suitable for real-time implementation if the aircraft capabilities are known. For example, the evader needs to know the pursuer's acceleration capabilities in addition to its own and vice versa. It is not difficult to verify that these feedback strategies constitute a saddle point for the minimax problem. Moreover, one can verify the optimality of U and W considered as open-loop strategies, (see Ref. 10 for further details).

It is known[10] that if the acceleration parameters of the pursuer a_i were smaller than the acceleration parameters of the evader b_i, the evader can escape for initial conditions away from the capture conditions. On the other hand, if the pursuer has a higher acceleration capability, then capture is assured for sufficiently large time-to-go. If these constants were different for different regions of the state-space, it is clear that the pursuer should attempt to force the evader to those regions of the state-space where it has a superior acceleration capability when compared with the evader. An opposite strategy should be followed by the evader. These strategies, however, can appear in the present formulation only if the matrices A and B were modeled to be explicit functions of the state variables. Although this does not appear feasible, an alternate possibility for handling the change in aircraft capabilities as functions of states is to store the A and B matrices at several flight conditions and use the instantaneous values for the remaining time-to-go. In any event, it is important to select the acceleration constraint matrices A and B, keeping the realistic capabilities of the aircraft involved in view. Otherwise, the controls emerging may not be implementable. In the next section, the pursuit-evasion guidance law derived here will be transformed to the original coordinates to obtain the nonlinear feedback law.

Several variations of the differential game discussed in this section are feasible. For example, it is known that for sufficiently large t_{go}, the feedback gains can become unbounded. To correct this situation, one may append the term $e^{Qt_{go}}$ to the integrand of the performance index.[16] The influence of this term will be to introduce a degree of insensitivity in the feedback strategies for large time-to-go. The constant Q in the exponent can be selected based on the desired degree of insensitivity. Modification of the present guidance law for the sampled data case is another interesting possibility. Additionally, one could impose saturation constraints on the acceleration components[17] and terminal aspect angle constraints[14] in this pursuit-evasion problem. The effect of imperfect information about the adversary's acceleration capabilities on the outcome of the game can also be studied in the present setting.

Nonlinear Feedback Strategy

The short-range pursuit-evasion problem between two aircraft was analyzed in the previous section using prelinearized models. The feedback solutions obtained there minimize the terminal miss subject to quadratic acceleration constraints. Since the prelinearizing transformation did not alter the position state variables in the original problem, the definition of terminal miss in the transformed and untransformed problems remains the same. This fact is important to ensure the usefulness of the solution. In order to make the guidance laws given in expressions (40)–(45) implementable, they may next be transformed back to the original coordinates using the expressions (17)–(22) as follows.

Define the quantities:

$$d_0 = K_1(1,1)(x_p - x_e) + K_2(1,1)(\dot{x}_e - \dot{x}_p) \tag{46}$$

$$d_1 = K_1(2,2)(y_p - y_e) + K_2(2,2)(\dot{y}_e - \dot{y}_p) \tag{47}$$

$$d_2 = K_1(3,3)(h_p - h_e) + K_2(3,3)(\dot{h}_e - \dot{h}_p) \tag{48}$$

$$f_0 = d_0 K_3(1,1) \quad (49)$$

$$f_1 = d_1 K_3(2,2) \quad (50)$$

$$f_2 = d_2 K_3(3,3) \quad (51)$$

with the terms $K_1(i,i)$, $K_2(i,i)$, $K_3(i,i)$, $i = 1, 2, 3$ being calculated from the expressions (42)–(44). The relative position and velocity components $(x_p - x_e)$, $(y_p - y_e)$, $(h_p - h_e)$, $(\dot{x}_p - \dot{x}_e)$, $(\dot{y}_p - \dot{y}_e)$, $(\dot{h}_p - \dot{h}_e)$ are the quantities measured by the pursuer and evader using onboard instrumentation. The expressions (46)–(51) may next be substituted in the expressions (17)–(22) to obtain the nonlinear pursuit-evasion guidance law as

$$\phi_p = \tan^{-1}\left[\frac{d_1 \cos\chi_p - d_0 \sin\chi_p}{\cos\gamma_p(d_2 + g) - \sin\gamma_p(d_0 \cos\chi_p + d_1 \sin\chi_p)}\right] \quad (52)$$

$$n_p = \frac{\cos\gamma_p(d_2 + g) - \sin\gamma_p(d_0 \cos\chi_p + d_1 \sin\chi_p)}{g \cos\phi_p} \quad (53)$$

$$T_p = [\sin\gamma_p(d_2 + g) + \cos\gamma_p(d_0 \cos\chi_p + d_1 \sin\chi_p)]m_p + D_p \quad (54)$$

$$\phi_e = \tan^{-1}\left[\frac{f_1 \cos\chi_e - f_0 \sin\chi_e}{\cos\gamma_e(f_2 + g) - \sin\gamma_e(f_0 \cos\chi_e + f_1 \sin\chi_e)}\right] \quad (55)$$

$$n_e = \frac{\cos\gamma_e(f_2 + g) - \sin\gamma_e(f_0 \cos\chi_e + f_1 \sin\chi_e)}{g \cos\phi_e} \quad (56)$$

$$T_e = [\sin\gamma_e(f_2 + g) + \cos\gamma_e(f_0 \cos\chi_e + f_1 \sin\chi_e)]m_e + D_e \quad (57)$$

From these expressions it can be seen that each participant in the differential game requires the differential positions and velocities in the x, y, h frame in addition to their own flight-path angle, flight azimuth, aerodynamic drag, and a thrust-throttle table. Moreover, the knowledge of the participant's acceleration capabilities and an estimate of time-to-go are required to compute the elements of the feedback gain matrices K_1, K_2, K_3. The need for estimating time-to-go may sometimes be perceived as a weakness of the present approach and those discussed in the literature.[9,10,13] However, it is to be noted that the pursuit-evasion game discussed in this paper can be reformulated by changing the independent variable from time to some other, more easily estimated, monotonic variable. A possibility that suggests itself here is the differential range. Examination of this and other possibilities are currently underway.

Results and Discussions

The pursuit-evasion guidance scheme derived in this paper is next mechanized on a three degree-of-freedom simulation of two aircraft. This simulation incorporated wing loading and the maximum thrust-to-weight ratio as the input parameters. The parameters used in the present work are representative of a few currently operational high-performance fighter aircraft. The lift coefficient C_L is first computed using the commanded load factor and the wing area. The drag is then interpolated from a table implementing the lift over drag (L/D) ratio given as a function of Mach number and lift coefficient. The time-to-go required in the calculations of guidance gains is computed using the formula

$$t_{go} = \frac{R - R_w}{|\dot{R}|} \quad (58)$$

at each integration step. In the expression (58), R is the differential range between the pursuer and evader, $\dot{R}$ the range rate, and R_w the weapon effectiveness range. Introduction of the variable R_w in (58) explicitly recognizes the fact that the pursuit-evasion game terminates when the evader is within the pursuer's weapon range. It is important to stress here that the present study did not account for the influence of terminal aspect angle on the weapon effectiveness range. Previously, Vergez[18] has noted that an accurate estimate of time-to-go is essential for realizing the full potential of optimal guidance laws. The method suggested in Ref. 18, however, is not acceptable for the differential game described here, because all the three position components are under control in the present situation.

The aircraft engagement simulation used a cubic spline interpolation scheme for the L/D ratio and a fourth-order Kutta-Merson integration technique. All the calculations were carried out with double precision arithmetic on a VAX 11/750 machine. In addition to the aircraft point mass model, first-order lags of the form

$$\dot{z} = k_c(z_c - z) \quad (59)$$

were included in the bank angle, load factor, and thrust channels to account for control surface and engine dynamics. In expression (59), z_c is the commanded value of the control variable, z the actual value, and k_c the actuator time constant. Note that these lags could have been explicitly handled in the guidance law derivation.

The speed brakes and thrust reversal were not implemented in the present simulation. Saturation limits were imposed on the engine thrust since it is not permissible to completely shut the engine off nor can the thrust be higher than the full afterburner value. Throughout the engagement, the engine thrust was not permitted to fall below the military power since there is a significant time constant associated with the engine spool windup and wind-down, often comparable to the engagement time. Both aircraft had the same T/W ratio, but the evader had a higher weight and wing loading. The L/D ratios for both aircraft were nearly the same. The acceleration capabilities of

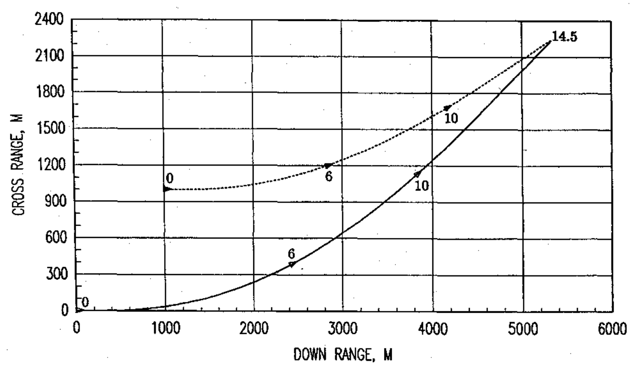

Fig. 1 Projection of pursuit-evasion trajectories in the horizontal plane—optimal interception (solid line: pursuer; dotted line: evader).

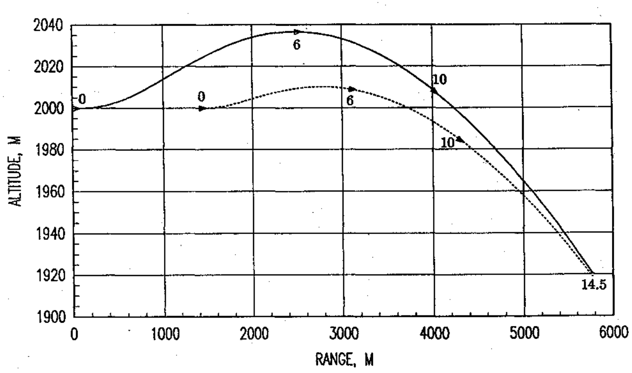

Fig. 2 Projection of pursuit-evasion trajectories in the vertical plane—optimal interception (solid line: pursuer; dotted line: evader).

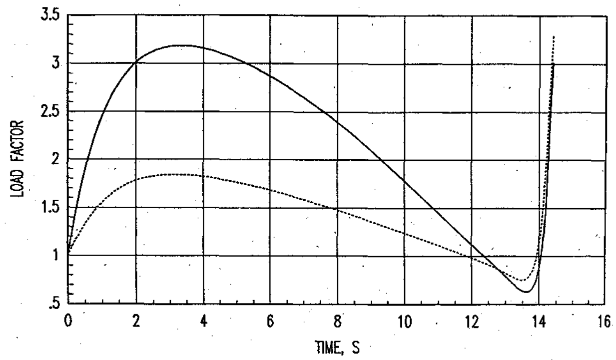

Fig. 3 Load factor histories for the pursuer and evader–optimal interception (solid line: pursuer; dotted line: evader).

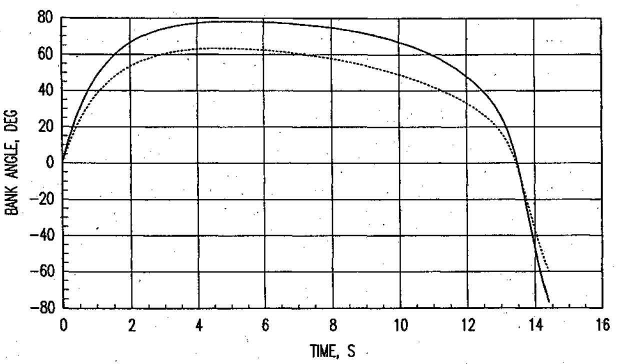

Fig. 4 Bank angle histories for the pursuer and evader–optimal interception (solid line: pursuer; dotted line: evader).

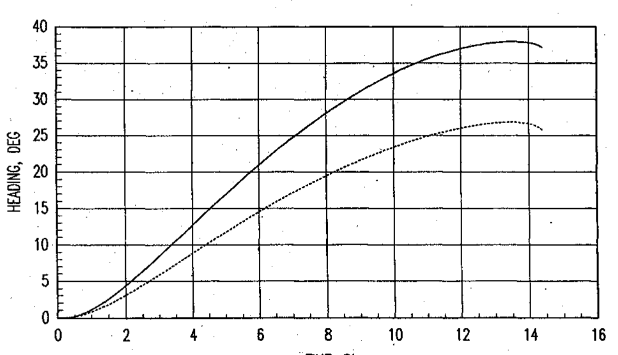

Fig. 5 Heading angle histories for the pursuer and evader–optimal interception (solid line: pursuer; dotted line: evader).

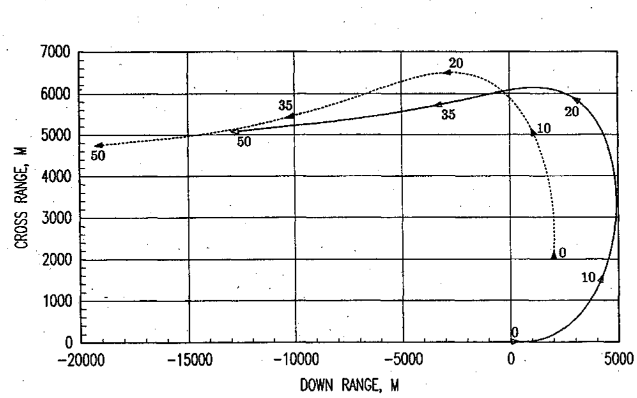

Fig. 6 Projection of pursuit-evasion trajectories in the horizontal plane–optimal evasion (solid line: pursuer; dotted line: evader).

the pursuer and evader required in the guidance gain computations were calculated using the expression (33) at the initial time. All the results given in the present study were generated with $R_w = 0$.

Although several computer runs were made, the results for two engagement scenarios will be given in this paper. The first scenario illustrates optimal interception by a pursuer at an initial offset of 1 km each in downrange and crossrange with the same initial heading as the evader. Both aircraft are at an altitude of 2000 m, with the evader speed at Mach 0.9 and the pursuer having about 100 m/s higher velocity. The evader attempts to avoid capture by veering to the left in the downrange crossrange plot given in Fig. 1. The evader climbs slightly before beginning to descend. The pursuer follows in a climbing turn before descending to capture the evader. Figure 2 shows the pursuit-evasion trajectory in the vertical plane. Capture occurs at about 5.8 km downrange with an elapsed time of about 14.5 s. A few time tics are provided in Figs. 1 and 2 to show the relative position of the vehicles as a function of time.

Close to the final time, the evader employs a pullout maneuver with about 3.25 g, which is nearly matched by the pursuer as can be seen from Fig. 3. This feature is reminiscent of the last-ditch maneuver sometimes employed by fighter pilots during a missile encounter. The bank angle histories for the pursuer and the evader are shown in Fig. 4. Throughout the engagement, the pursuer uses a higher bank angle than the evader. The heading angles for the scenario given in Fig. 5 indicate that the evader is continuously attempting to prevent the pursuer from matching the heading angle. Flight-path angle for the engagement shows a similar tendency. During this engagement, both aircraft use the military thrust since they are continuously attempting to improve their turn rates. It may be noted that if a nonzero weapon effectiveness range was included in this simulation, the pursuit-evasion game may have terminated sooner.

The second engagement scenario presented here is that of successful evasion. In this case, the pursuer and evader are initially offset by 2000 m in both downrange and crossrange. Again, at the initial time the pursuer and evader altitudes are equal at 2 km. The evader has an initial speed of 0.9 Mach, while the pursuer is at Mach 1.5. The pursuer's initial heading is 90 deg off that of the evader. The trajectories of both the pursuer and the evader in the horizontal plane are given in Fig. 6. From this figure it may be observed that the evader makes a 3.5-g initial turn before entering a dash, while the pursuer attempts to follow by making a tighter turn before dashing. By the time the pursuer completes the turn, it is too late and the evader escapes. The projection of the participant's trajectories in the vertical plane are given in Fig. 7. The apparent double-valued nature of the pursuer's trajectory is because it is in a turning descent. The relative vehicle position as the engagement evolves can be seen from the indicated time tic marks in the Figs. 6 and 7.

The evader and pursuer thrust histories indicate that the pursuer initially cuts the thrust to military power in order

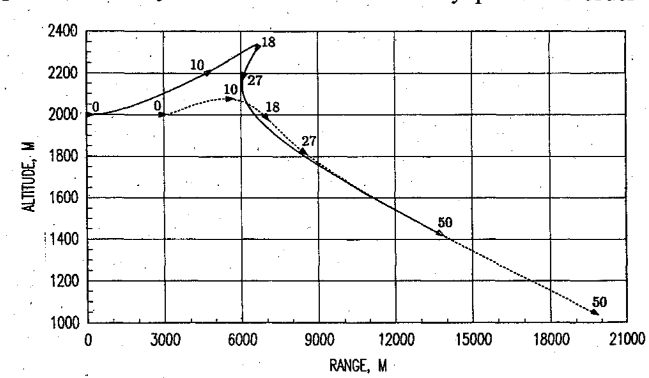

Fig. 7 Projection of pursuit-evasion trajectories in the vertical plane–optimal evasion (solid line: pursuer; dotted line: evader).

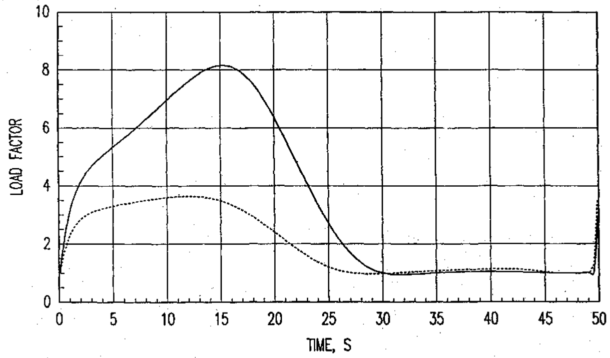

Fig. 8 Load factor histories for the pursuer and evader–optimal evasion (solid line: pursuer; dotted line: evader).

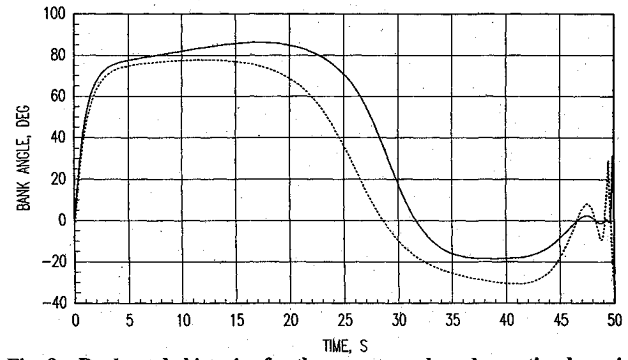

Fig. 9 Bank angle histories for the pursuer and evader–optimal evasion (solid line: pursuer; dotted line: evader).

to improve turning performance, and about halfway through the turn increases the thrust to full afterburner value to match the evader's speed. The evader uses full afterburner thrust throughout the engagement. The load factor histories for both aircraft are given in Fig. 8. The pursuer uses a load factor of 8 towards the end of its turn. This bleeds the pursuer's speed off to a value of lower than that of the evader. Since both aircraft have the same value of maximum T/W, their dash performance is nearly equal. As a result, the evader continues to maintain the separation between the two aircraft. From the bank angle histories given in Fig. 9, it can be seen that the participant's bank angles are close to each other. But, the pursuer is forced to use this large bank angle value for a longer duration than the evader. In the last five seconds of the game, the evader reverses its bank angle four times, which is matched by the pursuer. However, the bank angle excursions are of much smaller amplitude. Interestingly, this trajectory bears some resemblance to the weaving maneuver employed by fighter pilots during missile encounters.

The numerical results given in this section illustrate the performance of the nonlinear pursuit-evasion guidance law derived. Since the computational requirements are modest, this scheme should be implementable on state-of-the-art onboard computers. A more complete guidance law evaluation is currently underway.

At this juncture, it is important to stress that the present paper dealt with a pursuit-evasion differential game between two aircraft with preassigned roles. Recent literature[19,20] has noted that the pursuit-evasion model with a priori role assignment is inadequate for aerial combat game analysis. The suggested approaches include real-time role selection/switching based on the knowledge about the outcome of a set of pursuit-evasion games, and the pilot's willingness in assuming the risk of capture. Study of such aspects of aerial combat using the present pursuit-evasion solution will be of future interest.

Conclusions

This paper presented the development of nonlinear feedback strategies for aircraft pursuit-evasion. The research was aimed at the solution to the endgame problem wherein the pursuer attempts to minimize the terminal miss, which the evader attempts to maximize. The central ideas here were the use of linearizing transformations to cast the nonlinear pursuit-evasion problem into a linear, time-invariant form, obtaining its solution using the available differential game results, and transforming the guidance scheme back to the original coordinates to obtain implementable nonlinear feedback laws. For a specialized case, the solution was discussed in complete detail. Sample results for two aircraft with nearly equal acceleration capabilities were presented. The first set of results presented a successful interception, while the second set illustrated a successful evasion.

The present work is directly applicable for bank-to-turn missile guidance against maneuvering targets. Further, it can be used for deriving aircraft guidance schemes for collision avoidance, aerial rendezvous, and formation flight.

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